DERIVATION OF THE FORMULA ON P. 96, VOL. III, VIZ.;

$$R = r\times \frac{(n+1)N+(n-1)r^n}{(n-1)N+(n+1)r^n},$$

where r is an approximate value of $\sqrt[n]{N}$ and R a much nearer approxima'n.

Let $N = r^n + a$, then, by the binomial formula,

$$N^{\frac{1}{n}} = r\left(1+\frac{a}{nr^n}-\frac{(n-1)a^2}{1.2.n^2r^{2n}}+\frac{(n-1)(2n-1)a^3}{1.2.3.n^3r^{3n}}-\text{\&c.}\right).$$

Beginning with the term $a \div nr^n$ and reducing to a continued fraction and stopping at the second term of the cont'd fract. gives approximately

$$\frac{\dfrac{a}{nr^n}-\dfrac{(n-1)a^2}{1.2.n^2r^{2n}}+\text{\&c.}}{1} = \cfrac{1}{\cfrac{nr^n}{a}+\cfrac{1}{\cfrac{2}{n-1}+\text{\&c.}}} = \frac{2a}{2nr^n+(n-1)a};$$

$$\therefore R = r\left(1+\frac{2a}{2nr^n+(n-1)a}\right) = \frac{2nr^n+(n+1)a}{2nr^n+(n-1)a}.$$

Substituting for a its value $= N - r^n$,

$$R = r\times\frac{(n+1)N+(n-1)r^n}{(n-1)N+(n+1)r^n} = N^{\frac{1}{n}} \text{ nearly.}$$

R. J. ADCOCK.

SOLUTIONS OF PROBLEMS IN NUMBER SIX, VOL. IX.

SOLUTIONS of problems in No. 6, Vol. IX, have been received as follows:

From Florian Cajori, 422; Geo. E. Curtis, 419, 421; Prof. H. T. Eddy, 420; Geo. Eastwood, 422; Prof. A. Hall, 420; Henry Heaton, 419, 420, 422; Charles V. Kerr, 419; E. H. Moore, Jr., 419, 422; Levi W. Meech, 418; Thos. Spencer, 419; M. Updegraff, 419.

Prof. J. W. Nicholson sent elegant solutions of prob's 411 and 416, but his letter was accidentally misplaced, hence they were not included in notice of solutions in No. 6.

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418. *By Levi W. Meech, A. M., Norwich, Conn.*—"Required to express Lagrange's Theorem in terms of Finite Differences, as far as practicable, instead of the usual differentials."

SOLUTION BY THE PROPOSER.

Let θ denote an auxiliary, such that Lagrange's Theorem may take the form of the definite integral:

$$Fx = Ft + \int_0^1 d\theta \left\{ 1+\theta\frac{d}{dt}eft + \frac{\theta^2 d^2}{1.2.dt^2}(eft)^2 + \dots \right\} eft.\frac{dFt}{dt}.$$

Compare ANALYST, Vol. III, pages 34, 38; and Boole's Finite Differences, page 18. Then, since $d \div dt = D = \log(1+\Delta)$;

$$Fx = Ft + \int_0^1 d\theta.\varepsilon^{\theta Deft}.eft.\frac{dFt}{dt} = Ft + \int_0^1 d\theta\,(1+\Delta)^{\theta eft} eft.\frac{dFt}{dt}.$$

Developing $1+\Delta$ by the binomial theorem, and then integrating with respect to θ, we obtain the required formula:

$$Fx = Ft + eft.\frac{dFt}{dt} + \frac{\Delta}{1.2}(eft)^2.\frac{dFt}{dt} + \frac{\Delta^2}{1.2.3}(eft)^2(eft - 1\tfrac{1}{2})\frac{dFt}{dt}$$
$$+ \frac{\Delta^3}{1.2.3.4}(eft)^2(eft-2)^2\frac{dFt}{dt} + \dots$$

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419. *By C. E. Everett, Spirit Lake, Iowa.*—"Find the locus of a point starting from the centre of a given circle and moving so that the arc included between any two positions of the point shall equal the are of the circle intercepted by the radii drawn through the same positions."

SOLUTION BY GEO. E. CURTIS, BIRMINGHAM, CONN.

Let R be the radius of the given circle. The conditions of motion give at once the difierential equation of the locus,

$$ds = \sqrt{(r^2 d\theta^2 + dr^2)} = R d\theta;$$

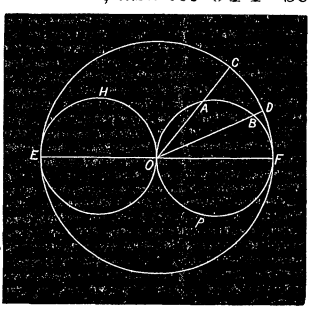
which by integration and reduction becomes

$$r = R \sin\theta.$$

This represents a circle half the radius of the given circle and internally tangent to it.

SOLUTION BY CHARLES V. KERR, ALLEGHENY, PENN'A.

Let O be the center of the given circle, EF a diameter, and let OPF be a circle described on the radius OF as diameter.



Through A and B, any two points on the circumf. of the inscribed circle, draw the radii OD and OC.

Now, a degree on the inscribed circle is equal in length to one-half of a degree on the given circle, since circumferences are to each other as their radii. But the are AB is double the measure of the insc'd angle AOB, and hence contains twice as may degrees as the are DC, which is the measure of the angle DOC; and since A and B are *any* two points the circle OHF is the req'd locus.

420. *By Prof. Asaph Hall.*—"Transform the definite integral

$$\int_b^a \varphi(x).dx,$$

so that the limits of integration shall be m and n."

SOLUTION BY HENRY HEATON, ATLANTIC, IOWA.

Put $x = py + q$. When $x = a$, $y = m$, when $x = b$, $y = n$. Hence we have $a = pm + q$ and $b\ pn + q$; whence

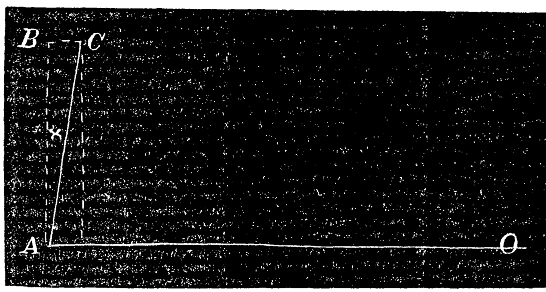
$$p = \frac{a-b}{m-n}, \text{ and } q = \frac{mb-an}{m-n};$$

$$\therefore \int_b^a \varphi(x)dx = p\int_n^m \varphi(py+q)dy = \frac{a-b}{m-n}\int_n^m \varphi\left(\frac{a-b}{m-n}y + \frac{mb-an}{m-n}\right)dy.$$

[Prof. Eddy and Prof. Hall obtain the same result as above, by a similar process, differing only slightly in the notation employed.]

421. *By George Eastwood, Saxonville, Mass.*—"In a Bicycle exercise on a level, circular course of given radius, what angle ought the plane of the machine to make with the vertical, so that the rider may move on the circumference of a perfect circle?"

SOLUTION BY FLORIAN CAJORI, MADISON, WISCONSIN.



Let $AO = r$ = radius of track; m = the mass; v = velocity; g = gravity, and x = the required angle. Also, let C denote the position of the centre of gravity of the bicycle and man, and put $a = AC$.

Then $r - a\sin x$ = radius of circumference described by C. The centrifugal force equals the mass into the square of the velocity divided by the radius, or $mv^2 \div (r - a\sin x)$.

Taking A as the origin of moments and equating the moment of the centrifugal force to the moment of grav., we obtain, after eliminating common factors, the equation

$$\frac{v^2}{r - a\sin x}\cos x = g\sin x,$$

wherein x is the required angle.

422. *By W. E. Heal, Marion, Indiana.*—"Determine the most general form of two algebraic functions φ and θ such that

$$\varphi(x)+\varphi(y)=\varphi[\theta(x,y)],$$

or prove that there are no such functions."

SOLUTION BY HENRY HEATON.

Put $\theta(x,y)=z$. Then if $\varphi(x)=ax+b$, we have $ax+b+ay+b=az+b$. Whence $z=a(x+y)+b$. If $\theta(x)=ax^2+bx+c$, we have $ax^2+bx+c+ay^2+by+c=az^2+bz+c$. Whence

$$z=-\frac{b}{2a}\pm\sqrt{\left[x^2+y^2+\frac{b}{a}(x+y)+\frac{b^2}{4a^2}+c\right]}.$$

If $\varphi(x)=ax^m+bx^{m-1}+cx^{m-2}+\dots+q$, we get the equation

$$az^m+bz^{m-1}+cz^{m-2}+\dots+q=a(x^m+y^m)+b(x^{m-1}+y^{m-1})+\dots+2q.$$

This equation can be solved in general terms only when m is less than 5.

SOLUTION OF PROB. 417, SEE P. 160, VOL. IX, BY GEORGE EASTWOOD.

Let n denote the number of marbles and t the number of transfers. At the last transfer but one, suppose the No. in A's bag to consist of m yellow and $n-m$ white, and the No. in B's bag, of $n-m$ yellow and m white marbles. Then the game presents four possible cases as follows:

1. One yellow from A to B and one yellow from B to A; and the prob. that such transfer will occur is $(m\div n)\times(n-m)\div n$. And as there are $n-m$ whites, the probable total No. of white balls will be $m(n-m)^2\div n^2$.

2. One yellow from A to B and one white from B to A; and the prob. of such transfer is $mm\div nn$; and the No. of whites in this case being $n-m+1$, the probable total No. of whites will be $m^2(n-m+1)\div n^2$.

3. One white from A to B and one yellow from B to A. The prob. of this will be $(n-m)^2\div n^2$, the No. of whites will be $n-m-1$, and the prob. total No. of whites will be $(n-m)^2(n-m-1)\div n^2$.

4. One white from A to B and one white from B to A. The probability is $m(n-m)\div n^2$; the No. of whites remains $n-m$, and the probable total is $m(n-m)^2\div n^2$ The sum of these four prob. values is $1+(n-m)(n-2)\div n$.

Now at the beginning $m=0$; hence at the end of the first transfers, there are certainly $n-1$ white marbles in A's bag. At the end of the 2nd transfers the prob. No. is $1+(n-1)(n-2)\div n$; at the end of the 3rd it is $1+(n-2)\div n+(n-1)(n-2)^2\div n^2$, and at the end of the t^{th} transfers it is

$$\left(n-1\right)\left(\frac{n-2}{n}\right)^{t-1}+\left(\frac{n-2}{n}\right)^{t-2}+\left(\frac{n-2}{n}\right)^{t-3}+\dots\ \left(\frac{n-2}{n}\right)+1=S, \text{ suppose.}$$

If we make $(n-2)\div n=1-p$, we shall have $S=[(1-p)^t+1]\div p$.

If we put n and t each $= 50$, we find $p = \frac{1}{25}$, and $S = 45.38431$. Hence the No. of white marbles in B's bag at the end of the 50th transfers will be at least 4. If we make $S = 25$, we shall find $25p - 1 = (1-p)^t$, which gives $0 = (1-p)^t$, and $0 = (\frac{24}{25})^t$; but this can only happen when t is infinite. Hence, after an inf. No. of trans. B will find half of A's marbles in his bag.

PROBLEMS.

423. *By E. Millwee, Add-Ran College, Granbury, Texas.*—Given the hypothenuse of a right-angled triangle and the difference of the two lines drawn from the acute ang's to the centre of the ins'd circle, to find the triang.

424 *By Prof. De Volson Wood.*—Required the equation to the locus which is at a constant, internal, normal dist. from the four cusp epicycloid.

425. *Id.*—A cylinder rolls down the upper plane of a wedge, the wedge being upon a perfectly smooth horizontal plane; required the velocity of the cylinder when it shall have rolled a distance l on the plane.

426. *By William Hoover, A. M.*—Eliminate θ from the equations

$$(a+b)\tan(\theta-\varphi) = (a-b)\tan(\theta+\varphi), \tag{1}$$

$$a\cos 2\varphi + b\cos 2\theta = c. \tag{2}$$

427. *By Prof. J. W. Nicholson.*—C and D are two fixed points (1 dist. apart) on the line AB; show that the locus of a point P, moving in such a manner that $\angle PDB = n\angle PCB$, the origin being at C, is

$$\frac{y\sqrt{-1}}{x-1} = \frac{(x+y\sqrt{-1})^n-(x-y\sqrt{-1})^n}{(x+y\sqrt{-1})^n+(x-y\sqrt{-1})^n}.$$

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Signal Service Tables of Rainfall and Temperature Compared with Crop Produc'n [Professional Paper No. X.]. 4to. Washington. 1882.

Parallax of a Lyræ and 61 Signi. By Asaph Hall, Professor of Mathematics, U. S. Navy. 4to. 64 pp. Washington. 1882.

The Theory of the Gas Engine. By Dugald Clark. 16mo. 164 pp. New York: D Van Nostrand, Publisher. 1882. Price 50 cts.

The American Engineer, 182–184 Dearborn Street, Chicago, Ill. An illustrated Journal, Scientific and Practical, *Published Weekly.* Sub. Price for U. S. and Canada, $4 per year.

On the Spherical Triangle Proof of the Addition Equation in Elliptic Functions. By Professor William Woolsey Johnson. [Ext. from Q. Jour. of Pure and Ap'd Math., No. 72.]

Systems of Formulæ for the sn, cn, and dn of $u + v + w$. By Prof. W. W. Johnson. [Extracted from the Proceedings of the London Math. Soc. Vol. XIII, No. 186.]